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1988 J. Phys. A: Math. Gen. 21 L353

(http://iopscience.iop.org/0305-4470/21/6/006)

J. Phys. A: Math. Gen. **21** (1988) L353-L356. Printed in the UK

LETTER TO THE EDITOR

Bond-diluted rigid animals for central force model

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Received 21 December 1987

Abstract. An exact enumeration of rigid clusters is analysed using Padé approximants. The number of bond-diluted rigid clusters with n_b bonds is found to grow like $n_b^{-\theta_b}(2.165 \pm 0.005)^{n_b}$ with $\theta_b = 0.988$, a value different from that of site-diluted rigid animals.

Randomly diluted elastic networks have been investigated by many authors [1-6]. There are several models one can consider. The isotropic model is not rotationally invariant. However, Alexander [7] has pointed out that this model is important in some amorphous materials which are under internal stress. There are three other models which are rotationally invariant, namely, the central force model [1], the bond bending model [8] and the granular disc model [9]. In the central force model, at the percolation threshold, a cluster is not rigid. As a result, one has the somewhat unrealistic phenomenon that the rigidity percolation threshold at $p = p_{cen}$ is much higher than the percolation threshold at $p = p_c$. For the bond bending model, $p_{rigid} = p_c$ in two dimensions, but this equality does not necessarily hold in high dimensions. Some authors [10, 11] have observed the same unrealistic problem, which occurred in the central force model, in high dimensions. For the bond bending model, a linear chain is not rigid in three or higher dimensions. The bonds A and C of the configuration in figure 1 can be twisted without costing any energy. Thus, the rigidity percolation threshold in this model is higher than the percolation threshold in three dimensions. This is also seen in simulation in three dimensions [12]. In fact, the bond bending model can be viewed as adding a second-nearest-neighbour interaction to the central force model. If one adds third-nearest-neighbour interactions to the bond bending model, or adds second- and third-nearest-neighbour interactions to the central force model, it will have the same threshold as that of percolation for some lattices (e.g. honeycomb lattice) in three dimensions. In a cubic lattice, one needs an infinitely long-range interaction to make the rigidity percolation threshold coincide with percolation threshold. This is an appropriate model for which one can check the conjecture

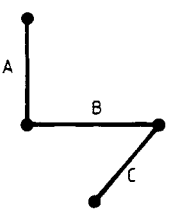

Figure 1. A cluster in three dimensions which is non-rigid for the bond bending model.

[5, 12-14] $f = t + 2\nu$ in three dimensions, where f is the bulk modulus exponent and t is the conductivity exponent. Generally in d dimensions, to make the thresholds coincide, one needs $(d(d-1)/2)$th nearest-neighbour interactions, one can consider beams [15] rather than bonds, or invoke a granular disc model which also needs $d(d-1)/2$ couplings in d dimensions. In the absence of such couplings, the threshold for the total rigidity is higher than that of percolation and some intermediate phase of partial rigidity is possible [16]. Up to now almost all the work which has been done involves bond-diluted problems. Thorpe and Garboczi [17] have considered the site-diluted elastic networks using effective medium theory. Recently Prunet and Blanc [18] have enumerated the rigid clusters for the central force model on a triangular lattice for both site dilution (saturated clusters in their nomenclature) and bond dilution. They found that, for the site-diluted problem, the fraction number of clusters of n_s sites which are rigid goes like $n_s^{1/2}\ (0.46 \pm 0.01)^{n_s}$.

In this letter, we enumerated the rigid clusters using a different method. We found that, for the bond-diluted problem, the approximate series obtained by [18] overestimates the exact result by more than 10% in higher order. We extrapolated the exponent governing the growth of rigid clusters using Padé approximants [19]. We also enumerated site-diluted rigid clusters and found the same series as [18].

For the bond-diluted central force model, a cluster with dangling bonds is non-rigid. Therefore, we first generated all possible boundaries of rigid clusters by enumerating one-loop diagrams embedded in the trangular lattice. Up to the order of p^{25}, one need not worry about possible internal boundaries. Based on a given one-loop diagram, we then generated all the rigid clusters. By a rigid cluster, we mean that the number of zero-frequency modes of the cluster is 3. Therefore, in the harmonic central force model, the diagram in figure 2 is excluded. This algorithm has enabled us to calculate the series for rigidity percolation [20] to the order p^{22}. Using this algorithm, we calculated the bond-diluted rigid clusters to the order p^{25} which took 10 CPU h on a VAX 11/785. From table 1 one can see that the error of the series in [18] is not within 1% for the higher-order term. This is because the relation in [18]

$$2n_s - n_b \leqslant 3 \tag{1}$$

where n_s is the number of sites in a cluster, n_b is the number of bonds in a cluster, is no longer true when $n_b \geqslant 15$. Figure 3 is a configuration with 15 bonds which violates equation (1) and is responsible for the difference in the coefficient of p^{15}.

If we assume that

$$a_{n_b} \sim n_b^{-\theta_b} \lambda_b^{n_b} \tag{2}$$

the series $\Sigma\, a_{n_b} p^{n_b}$ will behave as $(1 - \lambda_b p)^{\theta_b - 1}$. We use Padé approximants to determine θ_b. Table 2 shows the data for the Padé approximants from which we obtained

$$\lambda_b^{-1} = 0.462 \pm 0.001 \tag{3}$$

$$\theta_b = 0.988 \pm 0.001. \tag{4}$$

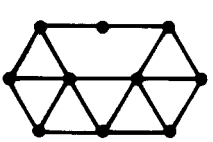

Figure 2. A configuration which is non-rigid for the harmonic central force model.

Table 1. The number of bond-diluted rigid clusters on the triangular lattice.

n_b	a_{n_b}	an_b [18]	n_b	a_{n_b}	a_{n_b} [18]	Deviation (%)
1	3	3	14	6	6	
2	0	0	15	448	460	3
3	2	2	16	27	27	
4	0	0	17	1 647	1 798	9
5	3	3	18	143	143	
6	0	0	19	6 219	7 235	16
7	6	6	20	687	687	
8	0	0	21	24 271	30 587	26
9	14	14	22	3 267	3 327	2
10	0	0	23	98 427	136 159	38
11	42	42	24	15 796	16 589	5
12	1	1	25	409 281		
13	135	135				

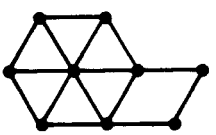

Figure 3. A configuration which violates equation (1).

Table 2. Dlog Padé estimates of λ_b^{-1} (and θ_b) for the bond-diluted rigid animals.

n	$[n-1/n]$	$[n/n]$	$[n+1/n]$
10		0.4618 (0.9857)	0.4624 (0.9880)
11	0.4626 (0.9879)	0.4619 (0.9884)	0.4618 (0.9884)
12	0.4618 (0.9884)		

We also analysed site-diluted rigid clusters using Padé approximants. We obtained

$$\lambda_s^{-1} = 0.43 \pm 0.01 \tag{5}$$

$$\theta_s = 0.52 \pm 0.01 \tag{6}$$

assuming that $a_{n_s} \sim n_s^{-\theta_s} \lambda_s^{n_s}$. So the number of site-diluted rigid clusters with n_s sites goes like $n_s^{-0.52} \lambda_s^{n_s}$. For the total number of clusters on a triangular lattice, one [21] has $\lambda = 5.183 \pm 0.001$ and $\theta = 1.00 \pm 0.01$ for site animals and $\lambda = 8.620 \pm 0.002$ and $\theta = 1.00 \pm 0.02$ for bond animals. If we make the ratio between rigid site animals and site animals, we obtain an exponent in agreement with the results of [18]. Here one sees that site and bond rigid animals have different exponents in contrast to site and bond total animals. It was unexpected that site and bond rigid animals should be in different universality classes. In view of our results, we would like to see whether the bulk modulus exponent for the site-diluted problem is different from the bond-diluted problem for the central force model.

In summary, we have calculated the exponent θ_b for bond-diluted rigid animals which is found to be different from that of site-diluted rigid animals.

I am grateful to Professor A B Harris for very helpful discussions and his critical reading of the manuscript. I thank the National Science Foundation, MRL program, for support and for computational facilities under grant no DMR 85-19059.

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